1. https://anonize.org

2. Introducing ANONIZE.

3. Open app, register using your UVA id

Love in the Time of Tindera

Gabriel García Marquez
We have a group of suitors and reviewers
Each has preferences over the other group.
We seek a stable matching between the two
Unstable Matching
Unstable Matching

G1 prefers 2 to 3

B2 prefers 1 to 2
Stable Matching
Stable matching has many practical applications
The National Resident Matching Program (NRMP) is a private, not-for-profit corporation established in 1952 to optimize the rank-ordered choices of applicants and program directors. The NRMP is not an application processing service; rather, it provides an impartial venue for matching applicants' and programs' preferences for each other consistently.

The first Main Residency Match (the Match) was conducted in 1952 when 10,400 internship positions were available for 6,000 U.S. graduating seniors. By 1973, there were 19,000 positions for just over 10,000 U.S. graduating seniors. Following the demise of internships in 1975, the number of first-year post-graduate (PGY-1) positions dropped to 15,700. The number of PGY-1 positions offered gradually increased through 1994 and then began to decline slowly until 1998. This year saw a record-high 26,678 PGY-1 positions offered (Figure 1), marking the twelfth consecutive annual increase in such positions.

The trend in the total number of applicants since 1952 is more dramatic, starting with 6,000 in 1952 and rising to a high of 36,056 in 1999. After a decline of 5,052 applicants from 1999 to 2003, the number of applicants has increased each year since the 2004 Match. Applicants registered for the 2014 Match reached an all time high of 40,394, an increase of 59 applicants over 2013.

For more information about the NRMP, please visit: www.nrmp.org. Additional data and reports for the Main Residency Match and the Specialties Matching Service (SMS) can be found at: www.nrmp.org/match-data. Instructions on how to request NRMP data also are provided.
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<th>Prior Year Graduates(^1)</th>
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University of Virginia
Chi Omega Bid Day 2012
Definition: matchings

\[ M = \{ m_1, m_2, m_3, \ldots, m_n \} \quad \text{suits} \]

\[ W = \{ w_1, w_2, w_3, \ldots, w_n \} \quad \text{reviewers} \]

\[ S = \{ (m_i, w_j) \}_{k} \quad \text{a set of pairs, including exactly one element from } M \text{ and one element from } W \]

* each \( m_i \in M \) appears in at most 1 pair
* each \( w_j \in W \)
Definition: matchings

Each $m_i (w_i)$ appears only one in a pairing.

A matching is perfect if every $m_i$ appears.

Each $w_i$ appears only one in a pairing.

$|S| = n$
Definition: preferences

\[ M = \{m_1, \ldots, m_n\} \]

Each element in \( M \) has preferences over \( W \).

\[ \forall i, \forall j, \forall k \in m_i \]

\[ W_i \leq_{m_i} W_j \leq_{m_i} W_k \leq_{m_i} W_2 \ldots \leq_{m_i} W_n \]
Example: preferences

\[ M = \{m_1, \ldots, m_n\} \]

\( m_i \) has a preference relation \( \prec_{m_i} \) on the set \( W \)

\( w_1 \prec_{m_i} w_4 \prec_{m_i} w_2 \prec_{m_i} w_8 \cdots w_n \)
\[(D, V) \quad (C, H)\]
$S = \begin{cases} 
\begin{pmatrix} 
\text{Dog} & \text{Cornell} \\
\text{Yale} & \text{IU} 
\end{pmatrix} 
& \begin{pmatrix} 
\text{Hippo} & \text{Harvard} \\
\text{Yale} & \text{UVA} 
\end{pmatrix} 
\end{cases}$

Consider this matching unstable match
Def: instability

\[ S = \left\{ \left( \begin{array}{c} w' \\ m' \end{array} \right), \left( \begin{array}{c} w \\ m \end{array} \right) \right\} \]

**INSTABILITY**: \( \exists \) an unmatched pair \((m^*, w^*) \notin S\) such that 
\((m^*, w) \in S\) \(\land\) \((m, w^*) \in S\), but 
\(w^* \succ m^*\) \(\land\) \(m \succ w^*\)
Def: instability

\[ S = \{ \begin{pmatrix} w' \\ \end{pmatrix}, \begin{pmatrix} m^*, w^* \end{pmatrix} \notin S \} \]

\[ w' \prec_{m^*} w^* \]
\[ m' \prec_{w^*} m^* \]
\[ M = \{ (s_1, r_1), (s_2, r_2), \ldots (s_n, r_n) \} \]
is a stable matching if

No unmatched pair \((s^*, r^*)\) prefer each other to their partners in \(M\)
Example 2

STABLE, even though some players are matched with their least favorite partners.
Prove: for every input there exists a stable matching.
proposal algorithm

Initialize all players to be unmatched

While $\exists$ an unmatched $m \in M$
  Let $w$ be the highest preferred element in $W$ that $m$
  has not yet asked.

If $w$ is unmatched, MAKEPAIR$(m,w)$

Else if $(m',w)$ is a pair AND $m \preceq m'$
  Break$(m',w)$
  MAKE$(m,w)$
StableMatch$(M, W, \prec_M, \prec_W)$

1. Initialize all $m, w$ to be free
2. while $\exists$FREE$(m)$ and hasn’t proposed to all $W$
   do Pick such an $m$
      Let $w \in W$ be highest-ranked to whom $m$ has not yet proposed
      if FREE$(w)$
         then Make a new pair $(m, w)$
      elseif $(m', w)$ is paired and $m' \prec_w m$
         do Break pair $(m', w)$ and make $m'$ free
         Make pair $(m, w)$
3. return Set of pairs
Proposal algorithm ends
Proposal algorithm ends

$O(n^2)$ steps

each $m$ proposes at most once to each $w$.
each $m$ proposes at most $n$ times.
size of $M$ is $n$. 

$\binom{n^2 - n}{n}$ steps 

$(10^5) \sim 10^{10}$
output is a matching

1. \( \text{Each} \ w \text{ is unmatched at the time it is matched in line } \text{IF ELSE}. \)

2. \( \text{Each} \ m \text{ is unmatched at time of proposal}. \)
output is perfect

If there is an unmatched \( m \) \( e \), then there is also an unmatched \( w \).

\[ \Rightarrow \text{ output has size } n. \]
if $\exists m$ who is free, then

$\exists w$ who has not been asked
output is stable

Suppose for the sake of reaching a contradiction that output has an instability. That means that there exists \((m^*, w^*) \neq S\) and 
\((m^*, w'), (m', w^*) \in S\) such that \(m' \leq m^* \quad w' \geq w^* \quad w \geq w^* \quad m \geq m^* \quad w \geq w^* \quad m \geq m^* \) But this contradicts

→ Consider the moment when \((m^*, w')\) was made into a match. Since \(w' \leq w^*\), then \(m^*\) must have already proposed to \(w^*\).

Either \(m^*\) proposed to \(w^*\) and \(w^*\) was in a match \((\hat{m}, w^*)\) or at a later point \(\hat{m}\) proposed and broke \((m^*, w^*)\) to form \((\hat{m}, w^*)\).

In either case, \(\hat{m} \geq m^*\). And either \(m^* = \hat{m}\) or

\(m^* \geq w^* \quad \hat{m} \geq w^* \quad m^*\)
output is stable

spse not.

\[ \exists (m^*, w), (m, w^*) \in S \quad w \prec_{m^*} w^* \quad m \prec_{w^*} m^* \]
output is stable

spse not. \exists (m^*, w), (m, w^*) \in S \quad w \prec_{m^*} w^* \quad m \prec_{w^*} m^*

m^* last proposal was to w
but \quad w \prec_{m^*} w^* \quad and so m^* must have already asked w^*
and must have been rejected by \quad m^* \prec_{w^*} m'

then either \quad m' \prec_{w^*} m \quad or \quad m' = m
which contradicts assumption \quad m \prec_{w^*} m^*
Proposer wins

\[(D, U) \quad (B, H) \quad \text{stable}\]

\[(B, U) \quad (D, H) \quad \text{stable}\]
Proposer wins
Remarkable theorem

w is valid for m:

\[ \text{best}(m): \{ w \mid w \text{ is valid for } m \} \]

\[ S^* = \{ (m, \text{best}(m)) \} \]

"Best matching for M"

GS returns \( S^* \). (no matter which order the proposals are made)
GS is Suitor-optimal.
GS matching vs R-opt
## Baseball elimination

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# Baseball elimination

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**Against**