3.01.2016

abhi shelat
what is the first rule of typesetting?

what did the variable $S_{ij}$ represent?

Express a recursive formula for $S_{ij}$:
It was the best of times, it was the worst of times, it was the age of wisdom, it was the age of foolishness, it was the epoch of belief, it was the epoch of incredulity, it was the season of Light, it was the season of Darkness, it was the spring of hope, it was the winter of despair, we had everything before us, we had nothing before us, we were all going direct to heaven, we were all going direct the other way - in short, the period was so far like the present period, that some of its noisiest authorities insisted on its being received, for good or for evil, in the superlative degree of comparison only.
It was the best of times, it was the worst of times, it was the age of wisdom, it was the age of foolishness, it was the epoch of belief, it was the epoch of incredulity, it was the season of Light, it was the season of Darkness, it was the spring of hope, it was the winter of despair, we had everything before us, we had nothing before us, we were all going direct to heaven, we were all going direct the other way - in short, the period was so far like the present period, that some of its noisiest authorities insisted on its being received, for good or for evil, in the superlative degree of comparison only.
First rule of typesetting

never print in the margin!

are simply not allowed
It was the best of times, it was the worst of times, it was the age of wisdom, it was the age of foolishness, it was the epoch of belief, it was the epoch of incredulity, it was the season of Light, it was the season of Darkness, it was the spring of hope, it was the winter of despair, we had everything before us, we had nothing before us, we were all going direct to heaven, we were all going direct the other way - in short, the period was so far like the present period, that some of its noisiest authorities insisted on its being received, for good or for evil, in the superlative degree of comparison only.
Second rule of typesetting

avoid big ugly whitespaces (slack)
Typesetting problem

input: \( W = \{w_1, w_2, w_3, \ldots, w_n\} \)

output: \( M_{\text{arg min}} \)

\( L = (w_1, \ldots, w_{\ell_1}), (w_{\ell_1+1}, \ldots, w_{\ell_2}), \ldots, (w_{\ell_x+1}, \ldots, w_n) \)

\[
c_i = \left( \sum_{j=\ell_i+1}^{\ell_i+1} |w_j| \right) + (\ell_{i+1} - \ell_i - 1)
\]

such that \( c_i \leq M \quad \forall i \)

\[
\min \sum (M - c_i)^2
\]

\[
\min \ (S_{\text{LACU}})^2
\]
imagine optimal

\[ \text{fwoll is } w_\ell \text{ slack when line starts with } w_\ell \]

\[ \text{BEST}_n = \text{BEST}_{\ell-1} + S^2_{\ell,n} \]
\[
\text{BEST}_n = \min \left\{ \text{BEST}_0 + S^2_{1,n}, \text{BEST}_1 + S^2_{2,n}, \text{BEST}_2 + S^2_{3,n}, \ldots, \text{BEST}_{\ell-1} + S^2_{\ell,n}, \ldots, \text{BEST}_{n-1} + S^2_{n,n} \right\}
\]
typesetting algorithm

make table for $S_{i,j}$

for $i=1$ to $n$

$$\text{best}[i] = \min\{ \text{best}[j] + s[j+1][i]^2 \}$$
typesetting algorithm

\[
\text{make table for } S_{i,j}
\]

for \( i = 1 \) to \( n \)

\[
\text{best}[i] = \min \{ \text{best}[j] + s[j+1][i]^2 \}
\]
how to compute $S_{i,j}$

slack when line
starts with $w_i$
and ends $w_j$
Simplest case

slack when line
starts with $w_i$
and ends $w_i$

$S_{i,c} = M - \{w_i\}$
Simplest case

slack when line starts with $w_1$ and ends $w_2$
how to compute $S_{i,j}$

$S_{i,i} = M - |w_i|$

$S_{i,j} = S_{i,j-1} - 1 - |w_j|$

slack when line starts with $w_i$ and ends $w_j$

If $S_{i,j} \geq M$, then set $S_{i,j} = 0$.
How to compute $S$

```
// compute $S_{ij}$
int S[][] = new int[n+1][n+1];
for(int i=1; i<=n; i++) {
    S[i][i] = M - lens[i];
    for(int j=i+1; j<=n; j++) {
        S[i][j] = S[i][j-1] - lens[j] - 1;
        if (S[i][j]<0) {
            while(j<=n) { S[i][j++] = infty; }
        }
    }
}
```
It was the best of times, it was the worst of times; it was the age of wisdom, it was the age of foolishness; it was the epoch of belief, it was the epoch of incredulity; it was the season of
first step: make $S_{i,j}$

\[ S_{i,i} = M - |w_i| \]

\[ S_{i,j} = S_{i,j-1} - 1 - |w_j| \]
first step: make $S_{i,j}$

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<td>9</td>
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<td>0</td>
<td>99</td>
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</tbody>
</table>

\[
\begin{array}{cccccccccccccccc}
2 & 3 & 3 & 4 & 2 & 6 & 2 & 3 & 3 & 5 & 2 & 6 & 2 & 3 & 3 & 3 & 3 & 2 & 7 & 2 & 3 & 3 & 3 \\
3 & 3 & 2 & 12 & 2 & 3 & 3 & 5 & 2 & 7 & 2 & 3 & 3 & 5 & 2 & 12 & 2 & 3 & 3 & 6 & 2
\end{array}
\]
second step: compute

\[ \text{Best}_1 = \min \left\{ \text{Best}_0 + (S_{1,1})^2 \right\} = 1600 \]

\[ \text{BEST}_i = \min_{j=0}^{i-1} \left\{ \text{BEST}_j + S_{j+1,i}^2 \right\} \]
second step: compute

\[
\begin{align*}
\text{best} & = \begin{bmatrix}
0 & 1600 & & & & & & & & & &
\end{bmatrix} \\
\end{align*}
\]

\[
\text{Best}_i =
\]

\[
\text{BEST}_i = \min_{j=0}^{i-1} \left\{ \text{BEST}_j + S_{j+1,i}^2 \right\}
\]

\[
\begin{array}{cccccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\
1 & 40 & 36 & 32 & 27 & 24 & 17 & 14 & 10 & 6 & 0 & 99 & 99 & 99 \\
2 & 39 & 35 & 30 & 27 & 20 & 17 & 13 & 9 & 3 & 0 & 99 & 99 &
\end{array}
\]
second step: compute

\[
\text{BEST}_i = \min_{j=0}^{i-1} \{ \text{BEST}_j + S^2_{j+1,i} \}
\]
$$\text{BEST}_i = \min_{j=0}^{i-1} \{ \text{BEST}_j + S_{j+1,i}^2 \}$$

<table>
<thead>
<tr>
<th>best</th>
<th>0</th>
<th>1600</th>
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It was
$$\text{BEST}_2 = \min \left\{ \frac{\text{BEST}_0 + (S_{1,2})^2}{\text{BEST}_1 + (S_{2,2})^2} \right\} \rightarrow 0 + 36^2 = 1296 \rightarrow 1600 + 39^2 > 2000$$
\[ \text{BEST}_i = \min_{j=0}^{i-1} \{ \text{BEST}_j + S_{j+1,i}^2 \} \]

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It was
\[
\text{BEST}_i = \min_{j=0}^{i-1} \left\{ \text{BEST}_j + S_{j+1,j}^2 \right\}
\]

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It was the
best i = \min_{j=0}^{i-1} \{BEST_j + S_{j+1,i}^2 \}

It was the best
It was the best of
\[
\text{BEST}_i = \min_{j=0}^{i-1} \{ \text{BEST}_j + S_{j+1, i}^2 \}
\]

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It was the best of times, it was the worst of times.

\[
\begin{align*}
\text{BEST}_{10} &= \min \left\{ \text{BEST}_{10} + (S_{10, 11})^2, \text{BEST}_9 + (S_{9, 11})^2, \ldots, \text{BEST}_0 + (S_{1, 11})^2 \right\} \\
&\rightarrow 0 + (40)^2 = 1600 \\
&\rightarrow 36 + (42 - 3)^2 = 36 + 3^2 = 36 + 156 = 192 \\
&\rightarrow 60^2
\end{align*}
\]
\[
\text{BEST}_i = \min_{j=0}^{i-1} \{ \text{BEST}_j + S_{j+1,i}^2 \}
\]

It was the best of times, it was the worst of times.

Best_{11} = \min \{ }
\begin{align*}
\text{BEST}_i &= \min_{j=0}^{i-1} \left\{ \text{BEST}_j + S^2_{j+1,i} \right\} \\
\text{BEST}_1 &= 0 \\
\text{BEST}_2 &= 1600 \\
\text{BEST}_3 &= 1600 \\
\text{BEST}_4 &= 1296 \\
\text{BEST}_5 &= 1024 \\
\text{BEST}_6 &= 729 \\
\text{BEST}_7 &= 576 \\
\text{BEST}_8 &= 289 \\
\text{BEST}_9 &= 196 \\
\text{BEST}_10 &= 100 \\
\text{BEST}_11 &= 36 \\
\text{BEST}_12 &= 0
\end{align*}

![Table](#)

It was the best of times, it was the worst of
\[ \text{best } i = \min_{j=0}^{i-1} \{ \text{BEST}_j + S^2_{j+1,i} \} \]

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<td>36</td>
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</table>

It was the best of times, it was the worst of times.

\[
\text{best } 11 = \min \left\{ \begin{array}{l}
\text{BEST}_{10} + S^2_{11,11} \\
\text{BEST}_9 + S^2_{10,11} \\
\text{BEST}_8 + S^2_{9,11} \\
\text{BEST}_7 + S^2_{8,11} \\
\text{BEST}_6 + S^2_{7,11} \\
\ldots
\end{array} \right\}
\]
It was the best of times,

it was the worst of
It was the best of times, it was the worst of times, it was

\[
B_{\text{BEST}_i} = \min_{j=0}^{i-1} \{B_{\text{BEST}_j} + S_{j+1,i}^2\}
\]

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<td>576</td>
<td>289</td>
<td>196</td>
<td>100</td>
<td>36</td>
<td>0</td>
<td>818</td>
<td>545</td>
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\[
B_{\text{BEST}_{13}} = \min \left\{ B_{\text{BEST}_{12}} + S_{13,13}^2, B_{\text{BEST}_{11}} + S_{12,13}^2, \ldots, B_{\text{BEST}_7} + S_{8,13}^2, B_{\text{BEST}_6} + S_{7,13}^2 \right\}
\]
It was the best of times, it
was the worst of times, it

\[
\text{BEST}_i = \min_{j=0}^{i-1} \{ \text{BEST}_j + S_{j+1,i}^2 \}
\]

\[
\begin{array}{ccccccccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\
\text{best} & 0 & 1600 & 1296 & 1024 & 729 & 576 & 289 & 196 & 100 & 36 & 0 & 818 & 545 \\
\text{choice} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 6 \\
\end{array}
\]
0 best: 0 ch 0
1 best: 1600 ch 0
2 best: 1296 ch 0
3 best: 1024 ch 0
4 best: 729 ch 0
5 best: 576 ch 0
6 best: 289 ch 0
7 best: 196 ch 0
8 best: 100 ch 0
9 best: 36 ch 0
10 best: 0 ch 0
11 best: 818 ch 6
12 best: 545 ch 6
13 best: 452 ch 7
14 best: 340 ch 7
15 best: 244 ch 8
16 best: 164 ch 8
17 best: 117 ch 9
18 best: 37 ch 9
19 best: 16 ch 10
20 best: 0 ch 10
21 best: 809 ch 14
22 best: 413 ch 15
23 best: 344 ch 15
24 best: 133 ch 17
25 best: 118 ch 17
26 best: 62 ch 18
27 best: 32 ch 19
28 best: 4 ch 20
29 best: 444 ch 23
30 best: 348 ch 23
31 best: 277 ch 24
32 best: 197 ch 24
33 best: 149 ch 24
34 best: 87 ch 26
35 best: 66 ch 26
36 best: 446 ch 31
37 best: 377 ch 31
38 best: 297 ch 32
39 best: 233 ch 32
It was the best of times, it was the worst of times, it was the age of wisdom, it was the age of foolishness, it was the season of Light, it was the season of Darkness, it was the spring of hope, it was the winter of despair, we had everything to lose, we had nothing to lose, it was a tale of two cities, it was the story of our lives.
Gerrymander
Map of Charlottesville Precincts and Polling Places

Instructions on Finding Your Street:
- Set the Zoom Level to 300%
- Find a Nearby Landmark (e.g. the 250 Bypass exit you frequently use)
- Zoom in Closer (400% - 500%)
- Follow Familiar Roads until you Find Your Street

Legend
Polling Place
- Herman Key Recreation Center
- Clark Elementary School
- Carver Recreation Center
- Walker Upper Elementary School
- Benjamin Tonsler Park
- Carter Family Life Center
- Venable Elementary School
- Alumni Hall
A4 P4 B4

py

A4

party

amount votes for B

z distib. 
gerrymander problem

given: $A_1, \ldots, A_n, \quad M$ people in it, all precincts have equal population

($(n \text{ is even})$)

an $A_i = \#$ for party $A$ in precinct $i$.

output: $D_1, D_2$. 2 districts, partition of the precincts.

\[ |D_1| = |D_2| \]

\[ A(D_1) > \frac{mn}{q} \]

\[ A(D_2) > \frac{mn}{q} \]

if possible majority for party $A$ in both districts.
gerrymander problem

given: \( m A_1, A_2, \ldots, A_n \) \( n \) is even

output: \( D_1, D_2 \)

such that \( |D_1| = |D_2| \)

\[
A(D_1) > \frac{mn}{4}
\]

\[
A(D_2) > \frac{mn}{4}
\]

or “failure” if no such solution is possible
example

\[ A_1 = 65 \]
\[ A_2 = 57 \]
\[ A_3 = 45 \]
\[ A_4 = 47 \]

\[ M = 100 \]

\[ A_1 + A_2 = 122 \]
\[ A_3 + A_4 = 92 \]
\[ A_1 + A_4 = 112 \]
\[ A_2 + A_3 = 102 \]
Imagine very last precinct and how it is assigned:

\[ D_1 \]
\[ x \text{ precincts} \]  
\[ k \text{ voters for } A \]  

\[ D_2 \]
\[ n-k-1 \text{ precincts} \]  
\[ y \text{ voters for } A \]  

\[ D_n \]
\[ k+1 \text{ precincts} \]  
\[ x+A_y \text{ voters} \]  

\[ D_1 \]
\[ k \text{ precincts} \]  
\[ x \text{ voters} \]  

\[ D_2 \]
\[ n-k \]  
\[ y + A_n \]
\( S_{j,k,x,y} = \begin{cases} 
\text{true} & \text{if } \left( \text{true} \right. \\
\text{false} & \text{otherwise.}
\end{cases} \)

Among the first \( j \) precincts, \( k \) precincts to \( D_1 \)
\( x \) vote for \( A \) in \( D_1 \)
\( y \) vote for \( A \) in \( D_2 \)

\( S_{j,k,x,y} = S_{j-1,k,x,A_j,y} \) \text{ or } \( S_{j-1,k-1,x,A_j,y} \)

"Assign \( P_j \) to \( D_1 \)"

"Assign \( P_j \) to \( D_2 \)"
\( S_{j,k,x,y} = \) there is a split of first \( j \) precincts in which \(|D_1|=k\) and 
\( x \) people in \( D_1 \) vote A 
\( y \) people in \( D_2 \) vote A
\[
gerrymander(P, A, m) \\
\text{initialize array } S[0,0,0,0] = \text{true} \\
S_{j,k,x,y} = S_{j-1,k-1,x-A_j,y} \lor S_{j-1,k,x,y-A_j} \\
S_{1,0,1,0} = S_{0,0,0,0}
\]
\[ S_{j,k,x,y} = S_{j-1,k-1,x-A_j,y} \lor S_{j-1,k,x,y-A_j} \]

gerrymander(P,A,m)

initialize array \( S[0,0,0,0] = 1 \)
for \( j=1,\ldots,n \)
for \( k=1,\ldots,n/2 \)
for \( x=0,\ldots,jm \)
for \( y=0,\ldots,jm \)
fill table according to equation
search for true entry at \( S[n,n/2, >mn/4, >mn/4] \)

\[ \Theta\left(\frac{n^2}{M^2}\right) \]

120s
\( M=10^6 \)